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Flavour symmetries and Kähler operators

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Abstract: Any supersymmetric mechanism to solve the flavour puzzle would generate mixing both in the superpotential Yukawa couplings and in the Kähler potential. In this paper we study, in a model independent way, the impact of the nontrivial structure of the Kähler potential on the physical mixing matrix, after kinetic terms are canonically normalized. We undertake this analysis both for the quark sector and the neutrino sector. For the quark sector, and in view of the experimental values for the masses and mixing angles, we find that the effects of canonical normalization are subdominant. On the other hand, for the leptonic sector we obtain different conclusions depending on the spectrum of neutrinos. In the hierarchical case we obtain similar conclusions as in the quark sector, whereas in the degenerate and inversely hierarchical case, important changes in the mixing angles could be expected.

Keywords: Supersymmetric Effective Theories, Quark Masses and SM Parameters, Supersymmetric Standard Model.

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1. Introduction

Family replication and flavour dynamics are among the most intriguing features of Particle Physics [1]. In the Standard Model, three generations of left-handed quarks, that transform as doublets under the weak interactions, form Dirac pairs with three generations of right-handed quarks, that transform as singlets. After the electroweak symmetry breaking, a 3×3 Dirac mass matrix arises, that is diagonalized by certain mass eigenstates. Experimentally, the mass eigenstates turn out to be mixtures of the weak eigenstates, which mixing is described by a 3×3 unitary matrix, the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2].

By observing different weak decay processes and using experiments of deep inelastic neutrino scattering, it has become possible to determine this mixing matrix with a fairly high accuracy [3]. The data indicate a hierarchy in the quark mixing angles, that can be conveniently emphasized using the Wolfenstein parametrization [4]

$$V_{CKM} = \begin{pmatrix} 1-\dfrac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\dfrac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)\,, \tag{1.1}$$

where λ is determined with a very good precision in semileptonic K decays, giving $\lambda \simeq 0.23$, and A is measured in semileptonic B decays, giving $A \simeq 0.82$. The parameters ρ and η are more poorly measured, but a rough estimate is $\rho \simeq 0.1$, $\eta \simeq 0.3$ [5]. In the Wolfenstein parametrization, the hierarchy arises from the increasing powers of the different elements in the small expansion parameter λ.

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The quark masses are also observed to be hierarchical, and this hierarchy can be expressed in powers of the same expansion parameter λ. Namely:

$$
\begin{aligned}
m_u : m_c : m_t &\sim \lambda^8 : \lambda^4 : 1\,, \\
m_d : m_s : m_b &\sim \lambda^4 : \lambda^2 : 1\,.
\end{aligned} \tag{1.2}
$$

The recent discovery and confirmation of neutrino oscillations has added a new perspective to the flavour puzzle [6]. Different experiments indicate that neutrino mass eigenstates are also flavour-mixed, resulting in a leptonic version of the CKM matrix, the Maki-Nakagawa-Sakata (MNS) matrix [7].[1] In stark contrast to the quark case, neutrino mixing angles are not small. Neutrino experiments point to a maximal or nearly maximal atmospheric angle (best fit value, $\sin^2\theta_{23} = 0.52$), a large solar angle (best fit value, $\sin^2\theta_{12} = 0.30$), and a small 13 angle ($\sin^2\theta_{13} \lesssim 0.053$ @ 3σ) [8]. On the other hand, our present knowledge of the leptonic masses has two opposite sides: whereas the charged-lepton masses are known with an astounding precision, and they also follow a hierarchical pattern,

$$
m_e : m_\mu : m_\tau \sim \lambda^{4-5} : \lambda^2 : 1\,, \tag{1.3}
$$

not much is known about neutrino masses. It is known that the atmospheric mass splitting is $\Delta m^2_{atm} \simeq 2.6\times 10^{-3}\,\mathrm{eV}^2$, and the solar mass splitting, $\Delta m^2_{sol} \simeq 6.9\times 10^{-5}\,\mathrm{eV}^2$. However, it is not known the actual mass spectrum (whether it is degenerate, hierarchical or inverse hierarchical) or the absolute scale of neutrino masses. In the hierarchical case one would have

$$
m_{\nu_1} : m_{\nu_2} : m_{\nu_3} \sim \lambda^x : \lambda : 1\;, \tag{1.4}
$$

with x arbitrarily high for $m_{\nu_1} \to 0$. Nothing is known either about CP violation in the leptonic sector.

It is indeed frustrating that despite the large amount of data available, a compelling theoretical scenario to explain these data is still lacking. In the Standard Model, fermion masses and mixing angles have their origin in the Yukawa couplings, that are parameters within this model. Therefore, the origin of flavour has to lie in the realm of physics beyond the Standard Model.

There are though some interesting proposals to explain the origin of these flavour patterns. Perhaps the most elegant is the Froggatt-Nielsen mechanism [9]: left-handed and right-handed quarks of different generations carry different charges under a flavour symmetry. This forbids the appearance of some Yukawas which are only generated through non-renormalizable operators (suppressed by some heavy scale M) that involve one or more scalar fields (φ), usually called flavons. When these scalar fields take a vacuum expectation value the family symmetry is broken spontaneously. Assuming that $\langle\varphi\rangle/M$ is of order λ then Yukawa couplings naturally small (in the 't Hooft sense) are generated, with a non-trivial pattern of masses and mixing angles depending on charge assignments.

As explained above, one of the assumptions is that the family symmetry breaking occurs at very high energies and involves superheavy fields. The presence of these superheavy

[1]With two extra phases when neutrinos are Majorana particles.

fields jeopardizes the lightness of the Higgs mass, and therefore supersymmetric versions of the Froggatt-Nielsen mechanism are more plausible than their non-supersymmetric counterparts. Many models along these lines exist in the literature, both for abelian [10]–[16] and for non abelian groups [17, 18, 19]. The addition of supersymmetry constrains even more the flavour pattern of Yukawa matrices: some entries that would be allowed in non-supersymmetric theories can now be forbidden by the holomorphicity of the superpotential (these are the so-called supersymmetric zeros).

A supersymmetric theory is defined by three functions: the superpotential, $W(\phi)$, the Kähler potential, $K(\phi, \phi^*)$, and the gauge kinetic functions, $f_a(\phi)$. Here, ϕ represents all the chiral matter superfields. Any supersymmetric mechanism for generating flavour would leave an imprint both in the superpotential, through the Yukawa couplings, and in the Kähler potential. In principle, both sources of flavour violation contribute to the CKM matrix. Taking just the superpotential, ignoring the non-trivial structure of the Kähler potential, one could compute the CKM matrix as the misalignment between the up and the down components of the quark doublet, when one goes from the weak eigenstate basis to the mass eigenstate basis. However, the supersymmetric theory also comprises the Kähler potential and should not be disregarded. The non-trivial structure of the Kähler potential would translate into non-canonical kinetic terms for quarks, and therefore these fields are non physical. The CKM matrix computed in this way is also non physical, and we will refer to it as the "naive" CKM matrix. The correct procedure requires the consistent redefinition of the quark superfields to render the kinetic terms canonical [20]. This procedure will yield the "physical" CKM matrix [21]. The purpose of this paper is to compare, in a model independent way, the physical CKM matrix and the "naive" one and study what implications follow.

In section 2 we derive a very useful and compact formula that relates the physical CKM matrix to the naive one. In section 3 we use that formula, taking full advantage of the hierarchy of quark masses, for a perturbative analysis of the physical CKM matrix and derive approximate formulas for the elements of this matrix in terms of the elements of the naive CKM matrix. In section 4 we illustrate our method with a particular example and discuss the physical interpretation of our results. In section 5 we consider the effects of canonical normalization in the leptonic sector. The conclusions are summarized in section 6.

2. The naive vs. the physical CKM matrix

The part of the superpotential relevant for quark mass matrices is

$$W_{\rm quark} = Y^u_{ij} Q_i u_j H_2 + Y^d_{ij} Q_i d_j H_1 \,, \tag{2.1}$$

where Q_i $(i = 1, 2, 3)$, denote the left-handed quark doublets, u_i (d_i) the up (down)-type right-handed quarks, and H_2 (H_1) the up (down)-type Higgs doublets. To fix the notation, the Yukawa couplings are diagonalized as

$$\begin{aligned} Y^u &= V_{u_L} D_u V^\dagger_{u_R} \\ Y^d &= V_{d_L} D_d V^\dagger_{d_R} \,. \end{aligned} \tag{2.2}$$

Disregarding the Kähler potential, one could compute the CKM matrix from the matrices that diagonalize the Yukawa couplings from the left. As explained in the introduction, this procedure is not complete in general, and the result would be the "naive" CKM matrix,

$$V_{CKM} = V_{u_L}^\dagger V_{d_L} \,. \tag{2.3}$$

To compute the physical CKM matrix, one has to tackle properly the Kähler potential. A general Kähler potential reads:

$$K = K^Q_{\bar{i}j} Q^{\bar{i}*} Q^j + K^u_{\bar{i}j} u^{\bar{i}*} u^j + K^d_{\bar{i}j} d^{\bar{i}*} d^j \,. \tag{2.4}$$

where the matrices K^ϕ (with $\phi = Q, u, d$) are dimensionless and hermitian (so that K is real). The minimal (or canonical) case corresponds to $K^\phi_{ij} = \delta_{ij}$. In general these matrices are functions of other chiral fields, e.g. the flavons, which might enter through non-renormalizable operators. If such fields take vacuum expectation values, or if $K^\phi_{ij} \neq \delta_{ij}$ already at the renormalizable level, non canonical kinetic terms would follow. Therefore, a superfield redefinition has to be performed in order to get to canonical kinetic terms. If K^Q is diagonalized as

$$K^Q = U_Q^\dagger D_K^Q U_Q \,, \tag{2.5}$$

where U_Q is unitary and D^Q_K diagonal, and similarly for K^u and K^d, then the redefined superfields

$$\begin{aligned} Q' &= [U_Q^\dagger (D_K^Q)^{1/2} U_Q] Q \equiv V_{Q'Q} Q \,, \\ u' &= [U_u^\dagger (D_K^u)^{1/2} U_u] u \equiv V_{u'u} u \,, \\ d' &= [U_d^\dagger (D_K^d)^{1/2} U_d] d \equiv V_{d'd} d \end{aligned} \tag{2.6}$$

give rise to a canonical Kähler potential $K = Q_i'^* Q'^i + u_i'^* u'^i + d_i'^* d'^i$. (Notice that we have defined the canonically normalized superfields such that $\phi' \to \phi$ as $D_K^\phi \to 1$, with ϕ any chiral field).

The corresponding superpotential written down in terms of the redefined superfields reads:[2]

$$W_{\rm quark} = Y'^{\,u}_{ij} Q'_i u'_j H_2 + Y'^{\,d}_{ij} Q'_i d'_j H_1 \tag{2.7}$$

where

$$\begin{aligned} Y'^u &= V_{QQ'}^T Y^u V_{uu'} \,, \\ Y'^d &= V_{QQ'}^T Y^d V_{dd'} \,, \end{aligned} \tag{2.8}$$

are the physical Yukawa couplings and

$$V_{\phi\phi'} \equiv V_{\phi'\phi}^{-1} = U_\phi^\dagger (D_K^\phi)^{-1/2} U_\phi \,. \tag{2.9}$$

[2] In our analysis we will ignore the effects of the canonical normalizations of the Higgs doublets, since their only effect is a flavour independent scaling of the Yukawa entries that does not affect the mixing angles.

These physical Yukawa couplings can be diagonalized from the left by V'_{u_L} and V'_{d_L}, and the misalignment between these two matrices yields the physical CKM matrix: $V'_{CKM} = {V'_{u_L}}^\dagger V'_{d_L}$.

It is not a trivial task to relate, in a model independent way, the "naive" CKM matrix to the physical one. To this end, we first rewrite eq. (2.8) as $Y'^u = Z_{u_L} D_u Z^\dagger_{u_R}$, and similarly for Y'^d. This looks like a singular value decomposition, but the matrices

$$\begin{aligned} Z_{u_L} &= V^T_{QQ'} V_{u_L} = [U^T_Q (D^Q_K)^{-1/2} U^*_Q] V_{u_L} \,, \\ Z_{u_R} &= V_{uu'} V_{u_R} = [U^\dagger_u (D^u_K)^{-1/2} U_u] V_{u_R} \,, \end{aligned} \tag{2.10}$$

are not unitary in general for $D^Q_K \neq I$. On the other hand, it is straightforward to construct unitary matrices from Z_{u_L} and Z_{u_R}, using the Gram-Schmidt procedure. Given the column vectors of the matrix $Z = (z_1, z_2, z_3)$, the procedure guarantees that the following set of column vectors is orthonormal:

$$\begin{aligned} w_3 &= \frac{z_3}{N_3} \,, \\ w_2 &= \frac{z_2 - \langle w_3, z_2\rangle w_3}{N_2} \,, \\ w_1 &= \frac{z_1 - \langle w_2, z_1\rangle w_2 - \langle w_3, z_1\rangle w_3}{N_1} , \end{aligned} \tag{2.11}$$

where N_1, N_2 and N_3 are normalization factors, and $\langle \, , \, \rangle$ denotes the scalar product in $\mathbb{C}^3$. (Subindices $u_{L,R}$ or $d_{L,R}$ should be understood everywhere in these formulas.) In matricial form, eqs. (2.11) can be cast as $Z = WT$, with W a unitary matrix with column vectors $W = (w_1, w_2, w_3)$ and T a lower triangular matrix:

$$T = \begin{pmatrix} N_1 & 0 & 0 \\ \langle w_2, z_1\rangle & N_2 & 0 \\ \langle w_3, z_1\rangle & \langle w_3, z_2\rangle & N_3 \end{pmatrix} . \tag{2.12}$$

Substituting these matricial forms of Z_{u_L, d_L} in Y'^u we get $Y'^u = W_{u_L} T_{u_L} D_u T^\dagger_{u_R} W^\dagger_{u_R}$. Finally, we diagonalize

$$T_{u_L} D_u T^\dagger_{u_R} = R_{u_L} D'_u R^\dagger_{u_R} \,, \tag{2.13}$$

with R_{u_L} and R_{u_R} unitary, to obtain

$$Y'^u = W_{u_L} R_{u_L} D'_u R^\dagger_{u_R} W^\dagger_{u_R} \,, \tag{2.14}$$

which is nothing but the singular value decomposition of the physical Yukawa coupling $Y'^u = V'_{u_L} D'_u {V'_{u_R}}^\dagger$, with $V'_{u_{L,R}} = W_{u_{L,R}} R_{u_{L,R}}$. Note that $V'_{u_{L,R}}$ are guaranteed to be unitary by construction. This diagonalization procedure in two steps has the advantage over the direct diagonalization $Y'^u = V'_{u_L} D'_u {V'_{u_R}}^\dagger$, that it will enable us to write down the physical CKM matrix in terms of the "naive" one in a simple way. Also, this factorization will prove to be useful later on, since the off-diagonal entries of R_u turn out to be suppressed by ratios of quark masses.

For quark masses, we can write a relation between the physical and naive Yukawa eigenvalues using the (non-singular, non-unitary) matrices

$$S_\phi \equiv R_\phi^\dagger T_\phi \,, \tag{2.15}$$

(where, as usual, $\phi = u_L, u_R, d_L, d_R$) in eq. (2.13). This relation is simply

$$\boxed{D'_u = S_{u_L} D_u S_{u_R}^\dagger} \tag{2.16}$$

and a similar one for the down quarks.

On the other hand, the physical CKM matrix is:

$$V'_{CKM} = {V'}^\dagger_{u_L} V'_{d_L} = R^\dagger_{u_L} W^\dagger_{u_L} W_{d_L} R_{d_L} \,. \tag{2.17}$$

The "naive" CKM matrix enters implicitly in the right-hand side of this equation. To make it explicit, we use $W^\dagger_{u_L} = W^{-1}_{u_L}$, $W = ZT^{-1}$ and eq. (2.10) to obtain

$$V'_{CKM} = R^\dagger_{u_L} T_{u_L} V_{CKM} T^{-1}_{d_L} R_{d_L} \,. \tag{2.18}$$

Making use again of the matrices S_ϕ defined in eq. (2.15) we find the central formula of the paper

$$\boxed{V'_{CKM} = S_{u_L} V_{CKM} S^{-1}_{d_L}} \tag{2.19}$$

Eqs. (2.16), (2.19) show explicitly that the effect of a non-canonical Kähler potential on quark masses and mixing angles is that of an equivalence transformation, as given by the matrices S_ϕ.

3. Perturbative analysis of the physical CKM matrix

All the formulas that we have presented so far are exact. To proceed we have to compute analytically the matrices R_{u_L,d_L} introduced in eq. (2.13). This cannot be done exactly, but as we will show explicitly later on, $(R_{d_L})_{12} \lesssim m_d/m_s \sim \lambda^2$, $(R_{d_L})_{13} \lesssim m_d/m_b \sim \lambda^4$, $(R_{d_L})_{23} \lesssim m_s/m_b \sim \lambda^2$, and similarly for the up-type rotations: $(R_{u_L})_{12} \lesssim m_u/m_c \sim \lambda^4$, $(R_{u_L})_{13} \lesssim m_u/m_t \sim \lambda^8$, $(R_{u_L})_{23} \lesssim m_c/m_t \sim \lambda^4$. Therefore, the smallness of the ratios of the quark masses allows a perturbative expansion of R_{u_L,d_L} and of the physical CKM matrix. At this point it is convenient to point out that the Gram-Schmidt procedure used in the previous section is not unique. The particular ordering of vectors used in eq. (2.11) is important for the success of the perturbative analysis carried out in this section because it starts by treating z_3 as a good approximation to the eigenvector with the heaviest eigenvalue (which is reasonable because corrections will be suppressed by the smallness of the other two eigenvalues), then proceeds with the next-to-heaviest eigenvector and so on.

We start computing the physical eigenvalues, by making the perturbative expansion of eq. (2.16). Dropping terms suppressed by ratios of quark masses, we obtain

$$\begin{aligned} m'_u &\simeq N_1^{u_L} N_1^{u_R} m_u \,, \\ m'_c &\simeq N_2^{u_L} N_2^{u_R} m_c \,, \\ m'_t &\simeq N_3^{u_L} N_3^{u_R} m_t \,, \end{aligned} \tag{3.1}$$

and similarly for the down-quark sector. Canonization of the Kähler potential just changes the masses by a normalization factor, that in many models is very close to one. A related conclusion from (2.16) is that if some quark is massless before taking into account Kähler corrections, it will remain so afterwards.

The computation of the physical CKM matrix is a bit more involved. We compute first the zero-th order approximation to V'_{CKM}, neglecting any effect coming from the matrices R_{u_L,d_L}, and then we discuss the effect of these matrices. The physical CKM matrix reads, at zero-th order

$$V'^{0}_{CKM} = T_{u_L} V_{CKM} T^{-1}_{d_L} \,. \tag{3.2}$$

Due to the triangular structure of the T matrices, the elements above the main diagonal of ${V'}^{0}_{CKM}$ have a fairly simple expression:

$$\begin{aligned}
(V'^{0}_{CKM})_{12} &= \frac{N_1^{u_L}}{N_2^{d_L}}(V_{CKM})_{12} - \frac{N_1^{u_L}}{N_2^{d_L}}\frac{\langle w_3^{d_L}, z_2^{d_L}\rangle}{N_3^{d_L}}(V_{CKM})_{13} \,,\\
(V'^{0}_{CKM})_{13} &= \frac{N_1^{u_L}}{N_3^{d_L}}(V_{CKM})_{13} \,,\\
(V'^{0}_{CKM})_{23} &= \frac{N_2^{u_L}}{N_3^{d_L}}(V_{CKM})_{23} + \frac{\langle w_2^{u_L}, z_1^{u_L}\rangle}{N_3^{d_L}}(V_{CKM})_{13} \,.
\end{aligned} \tag{3.3}$$

It is possible to compute explicitly the normalization factors, as well as the scalar products $\langle w, z\rangle$ in terms of the Kähler potential and the matrices that diagonalize the Yukawa couplings in the original superpotential, V_{u_L,d_L}. Defining $Q_\phi \equiv V_\phi^\dagger (K^{\phi T})^{-1} V_\phi$ for $\phi = u_L, d_L$ and $Q_\phi \equiv V_\phi^\dagger (K^{\phi})^{-1} V_\phi$ for $\phi = u_R, d_R$, and noting that $\langle z_i^\phi, z_j^\phi\rangle = (Q_\phi)_{ij}$ we obtain

$$\begin{aligned}
\langle w_2^\phi, z_1^\phi\rangle &= -\frac{1}{N_2^\phi}\frac{(Q_\phi^{-1})_{12}}{(Q_\phi)_{33}}\det Q^\phi \,, &\qquad (N_1^\phi)^2 &= \frac{1}{(Q_\phi^{-1})_{11}} \,,\\
\langle w_3^\phi, z_1^\phi\rangle &= \frac{(Q_\phi)_{31}}{N_3^\phi} \,, & (N_2^\phi)^2 &= \frac{(Q_\phi^{-1})_{11}}{(Q_\phi)_{33}} \det Q^\phi \,,\\
\langle w_3^\phi, z_2^\phi\rangle &= \frac{(Q_\phi)_{32}}{N_3^\phi} \,, & (N_3^\phi)^2 &= (Q_\phi)_{33} \,,
\end{aligned} \tag{3.4}$$

where ϕ represents $u_{L,R}$ and $d_{L,R}$.

Motivated by flavour symmetries, we find reasonable to assume that the Kähler is a perturbation from the identity. Hence, the diagonal elements of Q_ϕ are very close to one, and the off-diagonal elements are suppressed with respect to those in the diagonal. If this is the case, all the normalization factors are approximately one and all the scalar products are much smaller than one.[3]

From eq. (3.3) one realizes that, at zero-th order, the 13 angle in the CKM matrix does not change substantially, at most by a factor that is close to one. The equation for $({V'}^{0}_{CKM})_{13}$ also tells us that, barring cancellations, the 13 element of the "naive" CKM matrix, $(V_{CKM})_{13}$ should not be larger than $\sim \lambda^3 A$.

[3]Even in the case in which there are large mixings in V_{u_L}, as in democratic models [22], the off-diagonal elements of the matrix Q_ϕ are of the order of the perturbation itself.

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Concerning the 12 and 23 angles, since the observed values are respectively of order λ and $\lambda^2 A$, the $(V_{CKM})_{13}$ contribution in eq. (3.3), being at most of order $\sim \lambda^3 A$, is irrelevant. Therefore, to zero order in the perturbation expansion in ratios of quark masses, none of the CKM elements changes substantially:

$$
\begin{aligned}
(V'^0_{CKM})_{12} &\simeq \frac{N_1^{u_L}}{N_2^{d_L}} (V_{CKM})_{12} \, , \\
(V'^0_{CKM})_{13} &\simeq \frac{N_1^{u_L}}{N_3^{d_L}} (V_{CKM})_{13} \, , \\
(V'^0_{CKM})_{23} &\simeq \frac{N_2^{u_L}}{N_3^{d_L}} (V_{CKM})_{23} \, .
\end{aligned} \tag{3.5}
$$

Let us analyze now the first-order terms in the expansion. To this order, the off-diagonal elements in the rotation matrices R_{u_L} and R_{d_L} read:

$$
\begin{aligned}
(R_{d_L})_{12} &\simeq \frac{m_d}{m_s} \frac{N_1^{d_L}}{N_2^{d_L} N_2^{d_R}} \langle z_1^{d_R}, w_2^{d_R} \rangle \, , \\
(R_{d_L})_{13} &\simeq \frac{m_d}{m_b} \frac{N_1^{d_L}}{N_3^{d_L} N_3^{d_R}} \langle z_1^{d_R}, w_3^{d_R} \rangle \, , \\
(R_{d_L})_{23} &\simeq \frac{m_s}{m_b} \frac{N_2^{d_L}}{N_3^{d_L} N_3^{d_R}} \langle z_2^{d_R}, w_3^{d_R} \rangle \, ,
\end{aligned} \tag{3.6}
$$

and similarly for the up-quark sector. The explicit expressions for the scalar products and the normalization factors can be read from eq. (3.4). An order of magnitude estimate of these matrix elements is, in view of the observed hierarchy of masses,

$$
\begin{aligned}
(R_{d_L})_{12} &\lesssim \lambda^2 \, , \qquad & (R_{u_L})_{12} &\lesssim \lambda^4 , \\
(R_{d_L})_{13} &\lesssim \lambda^4 \, , \qquad & (R_{u_L})_{13} &\lesssim \lambda^8 \, , \\
(R_{d_L})_{23} &\lesssim \lambda^2 \, , \qquad & (R_{u_L})_{23} &\lesssim \lambda^4 \, ,
\end{aligned} \tag{3.7}
$$

since the $\langle w, z \rangle$'s in these equations depend on some off-diagonal terms of the Kähler potential, and are supposed to be smaller than one. Being the observed CKM angles of the order of λ, $\lambda^2 A$ and $\lambda^3 A(\rho - i\eta)$, it is apparent that the only rotations that could have some impact on the CKM angles are $(R_{d_L})_{12}$ and $(R_{d_L})_{23}$; the rest are smaller than $\sim \lambda^4$. Furthermore, all the remaining terms in the series expansion are suppressed at least by λ^4, and can be neglected.

To quantify the effect of these contributions we use eqs. (2.18), (3.2) to write

$$
V'_{CKM} = R^\dagger_{u_L} V'^0_{CKM} R_{d_L} \, . \tag{3.8}
$$

The 12 element in V'_{CKM} is of order λ, so the rotations in eq. (3.7) have only a subdominant effect. At the end of the day, the large size of the Cabibbo angle compared to the hierarchy of quark masses, implies that the dominant contribution to the quark flavour violation in the 12 sector must come from the Yukawa couplings, being the contributions from the Kähler potential subdominant. In other words, if one wants to compute the 12 angle, it is

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not necessary to normalize canonically the quark superfields: computing the Cabibbo angle from the initial Yukawa couplings, eq. (2.1), is going to give essentially the correct result:

$$(V'_{CKM})_{12} \simeq (V_{CKM})_{12}\,. \tag{3.9}$$

The case of the 13 angle, that experimentally is $(V'_{CKM})_{13} \simeq \lambda^3 A(\rho - i\eta)$ requires a more careful analysis. Following eqs. (3.8) and (3.6), we obtain:

$$(V'_{CKM})_{13} \simeq (V'^0_{CKM})_{13} + \frac{m_s}{m_b}[(V^\dagger_{d_R} K^d V_{d_R})^{-1}]_{23} (V'^0_{CKM})_{12}\,. \tag{3.10}$$

Note that rotations in the right-handed sector *do* contribute to the CKM matrix, although this contribution is suppressed by small ratios of quark masses. The second term in that formula is $\sim \lambda^3[(V^\dagger_{d_R} K^d V_{d_R})^{-1}]_{23}$ and only in the case in which the off-diagonal element of the Kähler potential is not very suppressed, this term could contribute substantially to the physical 13 angle (see below). In general, K^d departs from the identity through suppressed contributions of non-renormalizable origin and therefore there is an additional suppression through the off-diagonal element $[(V^\dagger_{d_R} K^d V_{d_R})^{-1}]_{23}$.

Finally, for the 23 angle, which experimentally is $(V'_{CKM})_{23} \sim \lambda^2 A$, we obtain:

$$(V'_{CKM})_{23} \simeq (V'^0_{CKM})_{23} + \frac{m_s}{m_b}[(V^\dagger_{d_R} K^d V_{d_R})^{-1}]_{23}\,. \tag{3.11}$$

The second term is $\sim \lambda^2[(V^\dagger_{d_R} K^d V_{d_R})^{-1}]_{23}$, and it could have some effect on the observed 23 angle, depending again on the Kähler potential for the down right-handed quarks. Nevertheless, if the off-diagonal terms of the Kähler potential are smaller than $\sim \lambda$, that term is at most $\sim \lambda^3$ and can also be neglected.

Let us consider also the case in which the Kähler potential is not a perturbation from the identity. As mentioned in the last paragraph, in this case the Kähler potential could contribute to the 23 angle. However, if this were the case, that same element of the Kähler potential would also contribute to the 13 angle, giving $(V'_{CKM})_{13} \simeq (V'_{CKM})_{23}(V'_{CKM})_{12} \simeq \lambda^3 A$, which is too large by a factor of three. Hence, the 23 angle has to come from the Yukawa couplings, otherwise the 13 angle would be too large. Note that the 13 angle cannot be made smaller by cancellations, since all the remaining terms are at least of order λ^4, or by radiative corrections, since these affect also the 13 angle in such a way that the relation $(V'_{CKM})_{13} \simeq (V'_{CKM})_{23}(V'_{CKM})_{12}$ is approximately scale independent [23].

From the above discussion, it is apparent that the 12 and 23 angles cannot have their origin in the Kähler potential. The 13 angle could, if $[(V^\dagger_{d_R} K^d V_{d_R})^{-1}]_{23}$ is not very suppressed. This would suggest some approximate symmetry between the second and third generations of right-handed down quarks. This symmetry could also generate Yukawa entries large enough to produce a 13 angle in the "naive" CKM matrix of the same order of magnitude, or larger, than the contribution from the Kähler potential. For instance, in the context of a U(1) symmetry, if the charges for d_{R2} and d_{R3} are identical, a large 23 entry in the Kähler potential is allowed by the symmetry. However, it is easy to check that if one wants to reproduce the Cabibbo angle and the correct ratios of quark masses, these charges imply a 13 element in the "naive" CKM matrix of $\sim \lambda^3$, and therefore as large as the contribution from the Kähler potential itself. In this very special case, the effects of the Kähler potential are also subdominant.

4. Froggatt-Nielsen symmetry

As we have mentioned already, supersymmetric Froggatt-Nielsen models can easily explain zeros in the Yukawa matrices by combining the flavour symmetry with the holomorphicity of the superpotential W. In this context, corrections to the Yukawa textures from the Kähler potential could play a very relevant role: couplings that were forbidden in W could be written in K with the final effect of lifting at some order the supersymmetric zeros of the Yukawa textures. Family symmetries that would have been otherwise excluded might be viable after all [12].

In fact, it is true that Kähler corrections can change significantly the Yukawa textures and lift some of the supersymmetric zeros in them. However, as we have shown in the previous section, the dominant effect comes from K^Q which is the same [by $\mathrm{SU}(2)_L$ gauge invariance] for u_L and d_L. This means that the change induced by the Kähler corrections affect the textures for Y^u and Y^d in a very correlated way and the net effect on V_{CKM} cancels out. The effects coming from K^u and K^d, the Kähler for the right-handed quarks, have to pay the price of a quark mass in order to propagate to the left-handed sector and this makes them subdominant.

We give now a concrete example, taken from [12], to illustrate this behaviour. It is a model with a $\mathrm{U}(1)_F$ flavour symmetry and a single flavon field φ, with F-charge -1. The F-charges of the quarks are such that $Q_{13} \equiv Q_1 - Q_3 = -2$, $Q_{23} = -3$, $u_{13} = 10$, $u_{23} = 7$, $d_{13} = 6$ and $d_{23} = 5$. With these charges the Yukawa textures are

$$Y^u \sim \begin{bmatrix} \lambda^8 & \lambda^5 & 0 \\ \lambda^7 & \lambda^4 & 0 \\ \lambda^{10} & \lambda^7 & 1 \end{bmatrix}, \qquad Y^d \sim \begin{bmatrix} \lambda^4 & \lambda^3 & 0 \\ \lambda^3 & \lambda^2 & 0 \\ \lambda^6 & \lambda^5 & 1 \end{bmatrix}, \tag{4.1}$$

with two supersymmetric zeros each. The naive CKM that follows is

$$V_{CKM} \simeq \begin{bmatrix} 1-\lambda^2/2 & \lambda & \lambda^8 \\ -\lambda & 1-\lambda^2/2 & \lambda^7 \\ -\lambda^8 & -\lambda^7 & 1 \end{bmatrix}, \tag{4.2}$$

with too small values of V_{ub} and V_{cb}.

For the Kähler matrices we have the textures

$$K^Q \sim \begin{bmatrix} 1 & \lambda & \lambda^2 \\ \lambda & 1 & \lambda^3 \\ \lambda^2 & \lambda^3 & 1 \end{bmatrix}, \qquad K^d \sim \begin{bmatrix} 1 & \lambda & \lambda^6 \\ \lambda & 1 & \lambda^5 \\ \lambda^6 & \lambda^5 & 1 \end{bmatrix}, \qquad K^u \sim \begin{bmatrix} 1 & \lambda^3 & \lambda^{10} \\ \lambda^3 & 1 & \lambda^7 \\ \lambda^{10} & \lambda^7 & 1 \end{bmatrix}. \tag{4.3}$$

Only K^Q, K^d_{12} and K^u_{12} are in principle large enough to be able to cure the smallness of V_{ub} and V_{cb}. In fact K^Q is able to fill in the zeros in Y^u and Y^d above and gives $Y'^{u,d}_{13} \sim \lambda^2$ and $Y'^{u,d}_{23} \sim \lambda^3$. The resulting textures, if uncorrelated, would have produced a CKM matrix much closer to the observed one, with $V'_{ub} \sim \lambda^2$ and $V'_{cb} \sim \lambda^3$. However, the correlation makes these matrix elements much smaller. Applying the general results of the previous section, eqs. (3.10), (3.11), we find that the Kähler contributions to V'_{ub} and V'_{cb} are of the same order of the naive ones, and thus too small:

$$V'_{ub} \simeq V_{ub} + \frac{m_s}{m_b}[(V^\dagger_{d_R} K^d V_{d_R})^{-1}]_{23} V_{uc} \sim \lambda^8 + \lambda^2[\lambda^5]\lambda \sim \lambda^8 \,, \qquad (4.4)$$

$$V'_{cb} \simeq V_{cb} + \frac{m_s}{m_b}[(V^\dagger_{d_R} K^d V_{d_R})^{-1}]_{23} \sim \lambda^7 + \lambda^2[\lambda^5] \sim \lambda^7 \,. \qquad (4.5)$$

Therefore, this $\mathrm{U}(1)_F$ symmetry has to be rejected (see also [16]).

In the previous example it might seem a coincidence that $[(V^\dagger_{d_R} K^d V_{d_R})^{-1}]_{23}$ happens to have just the size required to get a contribution of the same order as that of the naive CKM matrix. However there is a good reason for that. The simplest way of understanding it is that, by making holomorphic redefinitions of fields, there are operators in the Kähler potential that can be moved to the superpotential and viceversa. In fact, by such redefinitions one can make zero all the derivatives $\partial^n K/(\partial\phi^*_{i_1}\partial\phi_{i_2}\ldots\partial\phi_{i_n})$ $(n > 2)$ and their conjugates around a given point [24] (these are the so-called Kähler *normal coordinates*). For instance, in the example above, the 23 down-quark sector of the Kähler potential is of the form

$$K = |d_2|^2 + |d_3|^2 + \frac{1}{M^5}[d_2 d_3^* \varphi^5 + \text{h.c.}] \,. \qquad (4.6)$$

The holomorphic redefinition $d_2 \to d_2 - d_3\varphi^5/M^5$ transforms K into

$$K = |d_2|^2 + |d_3|^2 \left(1 - \frac{|\varphi|^{10}}{M^{10}}\right) . \qquad (4.7)$$

Such field redefinitions respect the flavour symmetry and therefore they do not change the texture of the Yukawas in the superpotential (which is supposed to be generic and to contain all couplings allowed by the flavour symmetry). To summarize, the K^d_{23} element cannot give contributions different from those already present through the Yukawa couplings because a holomorphic redefinition of fields can be used to move such operator from the Kähler to the superpotential,[4] supposed to be generic to start with.

5. The lepton sector

The analysis of the previous sections can be straight-forwardly extended to the lepton sector. We first discuss the case in which neutrino masses come from a non-renormalizable dimension five operator in the superpotential:

$$W_{lep} = Y^e_{ij} L_i e_j H_1 + \frac{1}{2}\kappa_{ij}(L_i H_2)^T (L_j H_2) \,, \qquad (5.1)$$

where L_i denote the left-handed lepton doublets and e_i the right-handed lepton singlets. The matrix κ, that has dimensions of mass^{-1}, produces neutrino Majorana masses in the lagrangian $(1/2)\mathcal{M}_\nu \nu^T \nu$ after electroweak symmetry breaking, where $\mathcal{M}_\nu = \kappa\langle H_2^0\rangle^2$. The notation for this section is parallel to the one we used in section 2: the charged-lepton

[4] If there are at least two flavon fields with charges of opposite signs, such holomorphic redefinitions to remove K^d_{23} might not be possible, but in such case, K^d_{23} itself will be more suppressed in general.

Yukawa coupling is diagonalized by $Y^e = V_{e_L} D_e V_{e_R}^\dagger$ and the dimension five operator by $\kappa = V_{\nu_L} D_\kappa V_{\nu_L}^T$. One could disregard the effects of the Kähler potential and compute the MNS matrix from the misalignment between the charged-lepton and neutrino components of the lepton doublet. This procedure would yield the "naive" MNS matrix, $V_{MNS} = V_{e_L}^\dagger V_{\nu_L}$.

The physical MNS matrix is computed by redefining properly the chiral superfields to bring the kinetic terms to their canonical form. Following the same steps as in section 2, eqs. (2.6)-(2.19), one obtains the following expresion for the physical MNS matrix in terms of the "naive" one:

$$V'_{MNS} = S_{e_L} V_{MNS} S_{\nu_L}^{-1}\,, \tag{5.2}$$

with $S_{e_L} \equiv R_{e_L}^\dagger T_{e_L}$ and a similar expression for S_{ν_L}.

We first compute the zero-th order in the expansion, i.e. we neglect any contribution coming from R_{e_L} and R_{ν_L}:

$$\begin{aligned}
(V'^0_{MNS})_{12} &= \frac{N_1^{e_L}}{N_2^{\nu_L}}(V_{MNS})_{12} - \frac{N_1^{e_L}}{N_2^{\nu_L}}\frac{\langle w_3^{\nu_L}, z_2^{\nu_L}\rangle}{N_3^{\nu_L}}(V_{MNS})_{13}\,,\\
(V'^0_{MNS})_{13} &= \frac{N_1^{e_L}}{N_3^{\nu_L}}(V_{MNS})_{13}\,,\\
(V'^0_{MNS})_{23} &= \frac{N_2^{e_L}}{N_3^{\nu_L}}(V_{MNS})_{23} + \frac{\langle w_2^{e_L}, z_1^{e_L}\rangle}{N_3^{\nu_L}}(V_{MNS})_{13}\,.
\end{aligned} \tag{5.3}$$

From these equations it is apparent that at order zero the 13 angle does not change substantially, and that, barring cancellations, the 13 element in the "naive" MNS matrix has to be of the same order of magnitude or smaller than the observed one (we recall that the 13 angle is bound to be less than ~ 0.23 at the 3σ level). On the other hand, the solar and atmospheric angles receive contributions from the 13 angle that are proportional to off-diagonal elements in the Kähler potential for the lepton doublets. It is interesting to note that if the solar angle is maximal in the "naive" MNS matrix, due for instance to some symmetry in the parameters of the superpotential, the contribution to the mixing from the Kähler potential could explain the observed deviation from maximality, provided the 13 angle is close to the experimental upper bound and K^L has a large off-diagonal entry in the 23 sector. Notice also that to guarantee that the observed atmospheric angle is close to the maximal value, as suggested by experiments, the off-diagonal entry in the Kähler potential in the 12 sector would have to be small (again barring cancellations).

The complete analysis of eq. (5.2), including the effects of the matrices R_{e_L} and R_{ν_L} is more involved. This requires the diagonalization of 3×3 matrices and, although can be done exactly, the resulting formulas are not very elucidating. In the quark case, we used the fact that quark masses are very hierarchical to simplify the formulas. However, the neutrino spectrum is not known yet. Three possibilities are allowed experimentally, degenerate, inverted hierarchy and normal hierarchy, and the analysis should be done separately for each case. We expect the analysis for the latter case to be similar to the quark case, and so would be the conclusions: the effects from the Kähler potential on the mixing angles would be at most of the same order of magnitude of the "naive" mixing angles themselves.

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On the other hand, we expect different conclusions for the case in which neutrinos are degenerate or inversely hierarchical. In this case, R_{ν_L} could have rather large off-diagonal entries that could contribute significantly to the mixing angles (depending of course on the structure of the Kähler potential). As a matter of fact, the sensitivity of neutrino mixing angles to extra effects when there are degeneracies in the neutrino mass spectrum is a very well known fact. For instance, when neutrinos are degenerate in mass, extra effects from radiative corrections can drive the mixing angles to values much different to the ones that one would naively deduce from the bare superpotential [25].

Here, we will discuss with some detail first the case in which neutrinos are hierarchical and then the degenerate case. In the hierarchical case, atmospheric and solar neutrino experiments indicate that the mass of the heaviest neutrino is approximately five times larger than the mass of the next-to-heaviest neutrino. This is a rather mild hierarchy, much milder than in the quark sector, and could have some impact on the mixing angles. On the other hand, the mass of the lightest neutrino is unknown — experiments are even compatible with a massless lightest neutrino. For simplicity, we will assume that the mass hierarchy between the lightest neutrino and the others is very large.

Under these assumptions, only $(R_{\nu_L})_{23,32}$ can give a sizeable contribution to the leptonic mixing angles; the remaining contributions are negligible due to the hierarchy in the charged-lepton sector and the assumed hierarchy between the lightest neutrino and the other two neutrinos. Consequently, to first order in perturbation theory, the physical mixing angles read

$$(V'_{MNS})_{12} \simeq (V'^0_{MNS})_{12}\,, \tag{5.4}$$

$$(V'_{MNS})_{13} \simeq (V'^0_{MNS})_{13} + \frac{m_{\nu_2}}{m_{\nu_3}}[(V^\dagger_{\nu_L} K^\nu V_{\nu_L})^{-1}]_{23} (V'^0_{MNS})_{12}\,, \tag{5.5}$$

$$(V'_{MNS})_{23} \simeq (V'^0_{MNS})_{23} + \frac{m_{\nu_2}}{m_{\nu_3}}[(V^\dagger_{\nu_L} K^\nu V_{\nu_L})^{-1}]_{23}\,. \tag{5.6}$$

The solar angle at first order does not differ substantially from the value at zero-th order. How the zero-th order solar angle is affected by the Kähler potential was discussed after eq. (5.3). On the other hand, the atmospheric angle is known to be very close to maximal: $\sin^2\theta_{23}$ lies in the interval 0.31-0.72 at the 3σ level, corresponding the best fit point to 0.52. This suggests the existence of some underlying symmetry in the superpotential that yields maximal atmospheric mixing in the "naive" MNS matrix. Under this assumption, the deviation of the atmospheric angle from the maximal value is controlled by $(m_{\nu_2}/m_{\nu_3})[(V^\dagger_{\nu_L} K^L V_{\nu_L})^{-1}]_{23}$. This combination also affects the physical 13 angle, and therefore, the contribution from the Kähler potential to the 13 angle is proportional to the deviation from maximality of the atmospheric angle:

$$(V'_{MNS})_{13} \simeq (V'^0_{MNS})_{13} + [(V'_{MNS})_{23} - (V'^0_{MNS})_{23}](V'_{MNS})_{12}\,. \tag{5.7}$$

From this formula one finds an interesting lower bound on the 13 angle:

$$\sin^2\theta_{13} \gtrsim \frac{1}{2}\sin^2\left(\theta_{atm} - \frac{\pi}{4}\right)\sin^2\theta_{sol}\,. \tag{5.8}$$

(We remind that this relation is valid under the assumption that the superpotential produces maximal atmospheric mixing by itself.)

In cases with some neutrino mass degeneracy (inverse hierarchy or degenerate) and making the reasonable assumption that the leptonic Kähler matrices K^L, K^e and K^ν can be written as perturbations of the identity, $K^\phi = I + \Delta^\phi$, we can make a different type of expansion, in first orden of the perturbations Δ^ϕ. The analysis (and the results) resemble those of renormalization group evolution in cases with degeneracy [25]. For the charged lepton mass eigenvalues we obtain (no sum in i)

$$m'_{e_i} \simeq m_{e_i}\left[1 - \frac{1}{2}(V_{e_L}^\dagger \Delta^L V_{e_L})_{ii} - \frac{1}{2}(V_{e_R}^\dagger \Delta^e V_{e_R})_{ii}\right], \tag{5.9}$$

while for the neutrino masses we get (no sum in i)

$$m'_{\nu_i} \simeq m_{\nu_i}\left[1 - (V_{\nu_L}^T \Delta^L V_{\nu_L})_{ii}\right]. \tag{5.10}$$

Once again we find that the corrected masses are proportional to the naive ones.

For the MNS matrix we find

$$V'_{MNS} \simeq V_{MNS} + X_e V_{MNS} - V_{MNS} X_\nu\,, \tag{5.11}$$

where X_e and X_ν are anti-hermitian with $(X_e)_{ii} = (X_\nu)_{ii} = 0$ and

$$\begin{aligned}(X_e)_{ij} &= \frac{1}{m_{e_j}^2 - m_{e_i}^2}\left[m_{e_i} m_{e_j} (V_{e_R}^\dagger \Delta^e V_{e_R})_{ij} + \frac{1}{2}(m_{e_i}^2 + m_{e_j}^2)(V_{e_L}^\dagger \Delta^L V_{e_L})_{ij}\right],\\ (X_\nu)_{ij} &= \frac{1}{2}\frac{m_{\nu_i} + m_{\nu_j}}{m_{\nu_j} - m_{\nu_i}}(V_{\nu_L}^T \Delta^L V_{\nu_L})_{ij}\,.\end{aligned} \tag{5.12}$$

It is straightforward to derive the second order corrections if they are needed. Here we simply notice from the first order result above that the corrections coming from the left-handed sector can indeed be large in cases of (near)-degeneracy, as anticipated and do not attempt a more detailed analysis.

To finish this section, let us discuss briefly the case in which the non-renormalizable dimension five operator in eq. (5.1) comes from a see-saw mechanism: $\kappa = Y^\nu \mathcal{M}^{-1} Y^{\nu T}$, where Y^ν is the neutrino Yukawa coupling and $\mathcal{M}$ is the right-handed Majorana mass matrix. It can be checked that the canonical normalization of the right-handed neutrino superfields does not affect the see-saw predictions for the low energy neutrino mass matrix (or the non-renormalizable operator κ) [16]. Therefore the analysis and results for this case are completely identical to the ones that we have just discussed for the non-renormalizable operator.

6. Conclusions

In this paper we have analyzed the effect of the Kähler potential on the Cabibbo-Kobayashi-Maskawa (CKM) matrix, taking into account the flavour violation that it induces on the Yukawa matrices when the kinetic terms are canonically normalized. We have derived an exact formula [eq. (2.19)] that relates the CKM matrix that one would naively compute from the Yukawa couplings, to the physical CKM matrix, computed redefining properly the quark superfields to render the kinetic terms canonical. We have analyzed this formula requiring only that the quark masses and mixing angles are the observed ones, and we have proved that the contributions to the CKM matrix from the Kähler potential are subdominant. Such subdominance has been found previously in some concrete models (see e.g. [10, 26]), but it could always be attributed to the particular properties of the model under discussion. In contrast, the proof we have presented in this paper is model-independent.

We have also undertaken a similar analysis for the lepton sector, to study the impact of the flavour mixing in the Kähler potential on the Maki-Nakagawa-Sakata matrix. Our conclusions are different depending on the neutrino mass spectrum. When neutrinos are hierarchical we do not expect the Kähler potential to be the dominant contribution to the mixing angles. At most they would give a contribution to the mixing of the same order of the contribution from the superpotential. This contribution could be important, though, to explain the deviations from maximality in the solar and atmospheric mixings, for the case in which the superpotential yields maximal mixing angles by itself. On the other hand, when neutrinos are degenerate, important changes in the mixing angles are expected, due to the sensitivity of the mixing angles to any new effects, when the superpotential yields degenerate mass eigenstates.

Note added. After the completion of this work, we learned about two groups working along the same lines [27, 28]. Their conclusions agree with ours, in the aspects where our analyses overlap.

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